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GENERALIZED REGULAR REGIME OF INHOMOGENEOUS EXOTHERMIC BODIES WITH FRAGMENTARY HEAT TRANSFER CONDITIONS

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The case of convective heat transfer of film-coated fragments of a body surface with media of different constant temperature is discussed. In the stage of a generalized regular regime the difference between the final and actual temperatures decreases in an inhomogeneous exothermic body in accordance with an exponential law.

The temperature field of an inhomogeneous convex being in a medium of constant temperature t_e is described by the Fourier equation [1-4]

$$\partial t/\partial\tau = a\nabla^2 t, \tag{1}$$

when the initial temperature distribution is known

$$t(P, 0) = t_0(P) \tag{2}$$

and convective heat transfer takes place over the body surface

$$\lambda \partial t(P_s \tau)/\partial n = \alpha [t_e - t(P_s, \tau)]. \tag{3}$$

Here λ is the thermal conductivity of the material, W/(m·K); α is the heat transfer coefficient, W/(m^2·K); t_e is the constant temperature of the surrounding medium, K; P_s are points of the body surface; ∂n is the element of the normal to the body surface at the point P_s. As shown by Boussinesq [5], the excess temperature of the body points, i.e., the difference between the actual temperature $t(P, \tau)$ and the medium temperature t_e, is expressed by the series

$$T(P, \tau) \equiv t(P, \tau) - t_e = \sum_{n=1}^{\infty} U_n(P) \exp(-m_n \tau), \tag{4}$$

which is arranged with respect to the exponential functions of time τ. In this case, the rates m_1 must satisfy the inequalities

$$0 < m_1 < m_2 < m_3 < \ldots \tag{5}$$

The latter circumstance has allowed Kondrat'ev [5, 6] and his group [7-10] to use an asymptotic representation of $t(P, \tau)$ in processing experimental data and in different calculations

$$T(P, \tau) = \sum_{n=1}^{\infty} U_n(P) \exp(-m_n \tau). \tag{6}$$

The distribution function $U_1(P)$, henceforth written without the subscript 1, is, as follows from ref. [1], a solution of the equation

$$\nabla^2 U + \frac{m}{a} U = 0, \tag{7}$$

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where a is the thermal diffusivity of the substance, m^2/sec, at the Fourier-Newton boundary condition [3]

$$\alpha U(P_s) + \lambda \partial U(P_s)/\partial n = 0 . \tag{8}$$

The rate m, sec^{-1}, is expressed in terms of the body surface S, m^2, its volume V, m^3, thermal diffusivity a, and characteristic dimension b, m; the Biot number and the uniformity criterion of a temperature field ψ are based on it as follows:

$$m = \frac{aS}{bv} \mathrm{Bi}\, \psi . \tag{9}$$

The uniformity criterion of a temperature field [11]

$$\psi = S^{-1} \int_S U(P_s)\, dS / v^{-1} \int_v U(P)\, dv \tag{10}$$

depends on the Biot number and the body geometry [12].

However, if the body is inhomogeneous, i.e., its heat capacity $C(P)$, $J/(m^3 \cdot K)$, and thermal conductivity $\lambda(P)$, $W/(m \cdot K)$ depend on the position of the point $P(x, y, z)$, and a stationary heat source $W(P)$, W/m^3, acts inside the body, then the temperature distribution $T(P, \tau)$ satisfies the Fourier-Poisson equation

$$C(P)\, \partial t/\partial \tau = \mathrm{div}\, [\lambda(P)\, \mathrm{grad}\, T] + W(P) . \tag{11}$$

Over each fragment S_g of the body surface S heat is transferred by convection to the medium with a constant temperature t_g. In accordance with the Newton law, on cooling the surface, i.e., when its temperature $t(P_g, \tau)$ is higher than that of the corresponding medium, a heat flux is described by the product

$$q_N(P_g, \tau) = \alpha_g [t(P_g, \tau) - t_g] \tag{12}$$

of fragmentary heat transfer coefficient α_g, $W/(m^2 \cdot K)$, and the temperature difference between the medium and the surface point $t(P_g, \tau)$. If a surface fragment is heated by the corresponding medium, i.e., its temperature t_g is higher than that of the points of the surface fragment $t(P_g, \tau)$, then the Newton flux is

$$q_N(P_g, \tau) = \alpha_g [t_g - t(P_g, \tau)] . \tag{13}$$

Usually [3] in cooling a heat flux is assumed positive and the heating flux, negative, although in numerous technologies the heat received by a workpiece is considered as positive. The surface fragments are coated with thin heat-conducting films whose thickness at each point is $b(P_g)$, m, and whose heat capacity is $C(P_g)$, $J/(m^3 \cdot K)$. These spacers are so then that the temperature gradient in them may be neglected and, on heating, they may be assumed to consume the power

$$q_P(P_g, \tau) = b(P_g),\ C(P_g)\, \partial T(P_g, \tau)/\partial \tau , \tag{14}$$

which constitutes the capacitive Paddy heat flux [2]. if a surface fragment is being cooled, i.e.,

$$\partial T(P_g, \tau)/\partial \tau < 0 , \tag{15}$$

then the capacitive Paddy flux is negative.

The conductive Fourier flux penetrates through the points P_g of the fragment S_g into the body

$$q_F(P_g, \tau) = \lambda(P_g)\, \partial t(P_g, \tau)/\partial n , \tag{16}$$

when the temperature gradient is positive, but if the temperature gradient is negative, then the heat flux is directed from the fragment points S_g to the surrounding medium

$$q_F(P_g, \tau) = -\lambda(P_g)\, \partial t(P_g, \tau)/\partial n . \tag{17}$$

The body itself and its film coating are heated by the power supplied to the body which is expressed by the Newton law (13). Obviously, on heating, the equation of heat balance at the points P_g will be as follows:

$$q_N(P_g, \tau) = q_F(P_g, \tau) + q_P(P_g, \tau) \tag{18}$$

If the body and its coating films are subjected to cooling, Eq. (18) still holds, but all terms on both sides of the equality acquire the opposite sign. Therefore in both cases the equation for energy conservation can be reduced to the invariant form

$$t(P_g, \tau) + \alpha_g^{-1} \lambda(P_g)\, \partial t(P_g, \tau)/\partial n + \alpha_g^{-1} b(P_g)\, C(P_g)\, \partial t(P_g, \tau)/\partial \tau = t_g, \tag{19}$$

which is valid for all heat conduction processes with the Fourier-Newton-Paddy boundary condition.

The temperature field described by Eq. (11) at high values of time is due independently of the initial distribution (2), to different stationary temperatures of the surrounding media t_g and has the following stationary limit:

$$\lim_{\tau \to \infty} t(P, \tau) = t_f(P) \tag{20}$$

which means the end of the heat conduction process inside the body with a stationary distributed heat flux and fragmentary heat transfer conditions over its surface. This final temperature field $t_f(P)$ satisfies the Poisson equation

$$\operatorname{div}[\lambda(P) \operatorname{grad} t_f] + W(P) = 0, \tag{21}$$

since together with the limit equality (20) the rate of temperature change tends to zero

$$\lim_{\tau \to \infty} \partial t(P, \tau)/\partial \tau = 0. \tag{22}$$

Obviously, instead of the boundary condition (19) the final temperature field satisfies the inhomogeneous Fourier-Newton condition

$$t_f(P_g) + \lambda(P_g)\, \alpha_g^{-1}\, \partial t_f(P_g)/\partial n = t_g, \tag{23}$$

because for the final field the capacitive Paddy flux (14) is absent. For a nonexothermic body that participates in heat exchange with the surrounding medium of constant temperature t_e, a final temperature will represent the temperature of this medium

$$\lim_{\tau \to \infty} t(P, \tau) = t_e \tag{24}$$

at all points of the body. Otherwise, for a body without internal heat sources located in a medium with a stationary temperature the final temperature distribution will be homogeneous, i.e., entirely gradientless. At fragmentary heat transfer conditions the existence of temperature field gradients is inevitable even in the absence of the internal heat source $W(P)$. Therefore the excess temperature, unlike [5, 6], is determined as a difference between the actual $t(P, \tau)$ and the final $t_f(P)$ temperatures

$$T(P, \tau) = t(P, \tau) - t_f(P). \tag{25}$$

Obviously, this excess temperature is a solution of the homogeneous Fourier equation

$$C(P)\, \partial T/\partial \tau = \operatorname{div}[\lambda(P) \operatorname{grad} T] \tag{26}$$

at the homogeneous Fourier-Newton-Paddy boundary condition

$$\alpha_g T(P_g, \tau) + \lambda(P_g)\, \partial T(P_g, \tau)/\partial n + C_g(P_g)\, b(P_g)\, \partial T(P_g, \tau)/\partial \tau = 0. \tag{27}$$

which is not related to the ambient temperatures. Integrating this relation with respect to the body surface and using, instead of (27), the Ostrogradsky formula yields the equation

$$\frac{d}{d\tau} \int_V C(P)\, T(P, \tau)\, dV + \int_S C(P_g)\, b(P_g) \frac{\partial T(P_g, \tau)}{\partial}\, dS = - \int_S \alpha_g T(P_g, \tau)\, dS. \tag{28}$$

Following Kondrat'ev [5, 6] and Boussinesq, we may, in a first approximation, represent the excess temperature $T(P, \tau)$ as the product of the distribution function $U(P)$ and the exponential function of time

$$T(P, \tau) = U(P) \exp(-m\tau). \tag{29}$$

The function of coordinates $U(P)$ and other thermophysical parameters of the substance, and of the heat transfer coefficients satisfies the Helmholtz equation

$$\operatorname{div} [\lambda(P) \operatorname{grad} U] + mC(P) U = 0 \tag{30}$$

and the modified (27) Fourier-Newton-Paddy condition

$$\alpha_g U(P_g) + \lambda(P_g) \frac{\partial U(P_g)}{\partial n} - mC(P_g) b(P_g) U(P_g) = 0. \tag{31}$$

In order to determine m, it is expedient to use Eq. (28), from which it follows that

$$m = \frac{\sum_g \int \alpha \sum_g \alpha_g \int_{S_g} U(P_g) dS_g}{\int_V C(P) U(P) dV + \sum_g \int_{S_g} b(P_g) C(P_g) U(P_g) dS_g}. \tag{32}$$

The quantity m is the complex distribution function of the fragments S_g over the body surface S, of heat capacity and thickness of the films, the body geometry, and the heat transfer coefficients. In the absence of the films on the body surface, constant values of the heat capacity of the substance, and of fragmentary heat transfer coefficients, the expression for rate (32) coincides with (9). It is sufficient to determine the mean values of the distribution functions over the surface fragments and in the body volume

$$\int_{S_g} U(P_g) dS_g = U_g S_g. \tag{33}$$

$$\int_V U(P) dV = U_v V, \tag{34}$$

and the mean heat capacities of the body

$$\int_V C(P) U(P) dV = \bar{C} U_v V \tag{35}$$

and of the films coating it

$$\int_{S_g} b(P_g) C(P_g) U(P_g) dS_g = [bc]_g U_g S_g, \tag{36}$$

so that the expression for rate (32)

$$m = \frac{\sum \alpha_g S_g U_g}{\bar{C} U_v V + \sum_g [bc]_g U_g S_g} \tag{37}$$

with constants

$$C(P) = \text{const}; \quad \lambda(P) = \text{const}; \quad \alpha_g = \alpha; \quad b = 0 \tag{38}$$

coincides with (9). If the volume heat capacity $C(P)$ can be neglected

$$C(P) \simeq 0, \tag{39}$$

then the cooling (heating) rate of the coating is

$$m = \sum_g \alpha_g U_g S_g \Big/ \sum_g [bc]_g U_g S_g . \tag{40}$$

It is pertinent to note that the steady-state ambient temperatures t_g and the internal heat source $W(P)$ do not influence the rate although they affect substantially the final temperature distribution $t_f(P)$. If these temperatures and the internal heat source are nonstationary but in the course of time are stabilized,

$$\lim_{\tau \to \infty} t_g(\tau) = t_g , \tag{41}$$

$$\lim_{\tau \to \infty} W(P, \tau) = W(P) \tag{42}$$

so that the relations

$$|W(P, \tau) - W(P)| < W_0 \exp(-m_1 \tau) , \tag{43}$$

$$|t_g(t) - t_g| < t_0 \exp(-m_2 \tau) , \quad \tau > \tau_r \tag{44}$$

and the quantities

$$m_1 , m_2 > m , \tag{45}$$

hold, then the final distribution satisfies the Poisson equation (21) and the boundary condition (23). Therefore the transition to the steady state will be described by the exponential relation (29). In case where the ambient temperatures and the internal heat source have no stationary stable limits, a relict component of the temperature field should not be forgotten whose major part decreases exponentially with time.

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